The Linearization Method Based on the Equivalence of Dissipated Energies for Nonlinearly Damped Structural Systems

Wenlung Li[*] and S. Tseng
Dept. of Mechanical Engineering, National Taipei University of Technology
[*] E-Mail: wlli@ntut.edu.tw

Abstract

Although the nonlinear damping characteristics inherently pertain to most structural systems, many useful dynamic models are still linear. As the consequences, a method that is able to equivalently linearize (EQL) the nonlinear damping so it can be directly applied in these existing linear models is essential. In the present paper, a novel EQL method for nonlinearly damped and single-degree-of-freedom systems has been developed. The method is derived by modulating the steady-state responses of the original nonlinear system. The current EQL requires the dissipated energy of the target linear system to equal to that of the original nonlinear one. As the result, this criterion is also equivalent to force both the phase angles and amplitudes of the two systems be equal or within a small allowable error. Furthermore, the paper has shown that to express the dissipated energy in terms of the Fourier coefficients of the modulated signal is possible. Thus, the equivalent viscous damping ratio can be computed from these Fourier coefficients. This new EQL method has been tested by the examples of bi-linear damping models. Both the results of the numerical simulations and the experimental data all verified the validity of the method. They also indicated that the method gives the equivalent viscous damping with good accuracy. In addition, a quality index that signifies how well the EQL system reaches is appropriately added.

Keywords: EQL, Nonlinear Damping, Viscous Damping, Fourier Coefficients.

1. Introduction

In most engineering practice, the use of a linear model for the application leads to fairly simple and useful results. And, there has been a comprehensive linear theory for treating problems of this type for quite a long time. Unfortunately, however, there exists no real problem is exactly linear. The easiest way is to assume that the system is almost linear or the nonlinearity is negligibly small in order to apply the linear theory. In general, nonlinearity of a system becomes more significant as vibration increases. As soon as the nonlinear effect is too large to be neglected, one is faced with the problem of predicting the responses of much harder nonlinear ones. Therefore, a natural idea of attacking nonlinear problems is to replace the nonlinear elements of systems by a linear one, which in some extend that having approximately equal responses, so that the linear theory can be reasonably applied.

Mathematically, any criterion of linearization shall preserve the characteristics of the original nonlinear system as many as possible [1]. There is no way to know the first inventor of the linearization method, as far as the authors' knowledge is concerned. However, almost all investigation reports in engineering practices concerning linearization directly or indirectly cited the paper from Krylov and Bogoliulov [2] of 1937, no matter they are in deterministic or stochastic systems. The asymptotic method given by Krylov-Bogoliulov was later known as the averaging method [2]. Since then, there have been a lot of different versions of linearization methods being developed. For example, the techniques for deterministic dynamics include like the perturbation methods [e.g., 3, 4], the harmonic balancing methods [e.g., 5] and the methods of multiple scales [6], etc. These methods are analytical rather than numerical and provide an alternative to computer solutions. In addition to these, there are also linearization methods that combining both analytic and numerical techniques. These algorithms include like [7] and the so-called phase-space linearization method [8]. The latter method is based on replacement of the nonlinear vector field by a set of linear ones, which are valid over a sufficiently small interval of time. Thus, it is actually a method that decomposes the nonlinear governing equation into a set of linear equations, each valid and evolving over a segment of trajectory. This linearization idea was later generalized and reported in [9].

Studies of linearization techniques for stochastic systems, on the other hand, significant amount of researches have been devoted in this area during the last couple decades. The pioneer works include [e.g., 10, 11]. Their method for stochastic nonlinear systems was later called statistical linearization [12], in which the nonlinear function of the original system is replaced by a linear function. In addition to the work from [13] which was later called equivalent linearization (EQL) [12] suggested replacing the nonlinear system by an equivalent one. Extensive literature review can be found, for example, Roberts and Spanos [14] or Socha and Soong [15]. Based on the linearization knowledge from sdof, it has been generalized into mdof systems. In order to solve dynamic system of large mdof, [e.g., 16-18] proposed their linearization methods together with taking the computational efficiency into their considerations. Besides, there are also a group of researchers developed linearization techniques by combining numerical methods. The elegant work [17] even employed the linearization idea to decompose the non-Gaussian distribution into a sum of Gaussian ones for statistical systems. By doing so, the method is then applicable to higher dimensional problems.

Amongst all these linearization methods, the system nonlinearity the most methods face with has been focused in that either localized at a small area or of stiffness. Only a few reports specifically mentioned the applicability of their methods on nonlinear damping. Apparently, damping or friction induces a lots of nonlinear dynamic phenolmena [1,15,19 and 20]. However, to the authors' knowledge, there exist only a few reports that discussed the linearization of this kind. The main reason may stem from that the damping of system is extremely difficult to accurately obtain. Generally, damping characteristics, no matter they are internal or external can be recognizable only by experimental measurements. As the result, only those linearization methods that physically incorporate with measuring signals can be capable of being used in engineering practices.

Though [21] has provided a new method to evaluate the system damping if it is linear. However, there is no way to clearly know the nature of nonlinear damping *a priori*.

2. Derivation of the Method

In general, a unit mass, sdof system with nonlinear damping can be represented together with initial conditions by

$$\ddot{z} + f_{\mathrm{d}}(z,\dot{z}) + \omega_{\mathrm{n}}^2 z = f(t)\,, \qquad (1)$$

where $f_{\mathrm{d}}(z,\dot{z})$ denotes the nonlinear damping function, ω_{n} the undamped circular natural frequency, and $f(t)$ the externally applied forcing function, respectively. If the system is excited by a harmonic function, without loss of the generality, one lets the forcing function be expressed as

$$f(t) = F\sin(\Omega t)\,. \qquad (2)$$

Thus, the solution of system (1) can be written as

$$z(t) = z_0(t) + z_{\mathrm{p}}(t)\,, \qquad (3)$$

in which $z_0(t)$ is the free or zero-input response and characterized by the system itself and the initial conditions, and $z_{\mathrm{p}}(t)$ the forced or zero-state response. Furthermore, $z(t) \approx z_{\mathrm{p}}(t)$ if only the steady-state responses are of the interest. It is also clear that the latter mainly depends on the characteristics of the excitation. Since the external forcing function is periodic, as shown in eq. (2), the total work done by/to the system (1) for the response per cycle can be represented by

$$\oint(\ddot{z} + f_{\mathrm{d}}(z,\dot{z}) + \omega_{\mathrm{n}}^2 z)\cdot dz = \oint f(t)\cdot dz\,, \qquad (4)$$

where the circle-integral denotes the integration of one complete cycle. Note that the expression on the RHS of eq. (4) actually represents the energy input to the system by the mechanism of the external forcing function $f(t)$. Because of the energy conservation between the kinetic and potential energies in each cycle, the total work done by them is zero. Thus, the averaged dissipated energy per radian can be evaluated from the LHS of eq. (4) and is

$$w_{\mathrm{d},z} = \frac{1}{2\pi}\oint f_{\mathrm{d}}(z,\dot{z})\cdot dz = \frac{1}{2\pi}\int_0^{T_1} f_{\mathrm{d}}(z,\dot{z})\cdot \dot{z}\, dt , \quad (5)$$

in which T_1 is the period of the $f(t)$. Notice from eq. (4) that $w_{\mathrm{d},z}$ is also equal to the averaged energy input to the system. Or, the loss of energy due to the damper is supplied by the excitation under the steady-state [22]. Since the external excitation is sinusoidal, it is reasonable to write the nonlinear responses as two parts: the time invariant and periodically alternating parts. In other words, one writes the steady state of $z(t)$ as

$$z(t) = Z_0 + z_a(t) = Z_0 + Z_a \sin(\Omega t - \phi) , \quad (6)$$

where ϕ is the phase angle which is mainly due to the nonlinear damping. Hence, one computes the input energy per radian from the RHS of eq. (4) and obtains

$$w_{\mathrm{d},z} = \frac{1}{2\pi}\int_0^{T_1} f(t)\dot{z}(t)\, dt = \frac{FZ_a}{2}\sin\phi , \quad (7)$$

after using eq.(2). Therefore, in order to evaluate the dissipated energy of a nonlinearly system, one may compute it from its input energy, or eq. (7), instead of directly computing from eq. (5).

Applying the input modulation concept given in [21, 23], or one modulates the steady-state of $z(t)$ by its input. Thus, the product modulation of $f(t)$ and $z(t)$ has the form

$$g_z(t) = f(t)\cdot z(t) = \frac{F\cdot Z_a}{2}\cos\phi + Z_0 F\sin(\Omega t) - \frac{F\cdot Z_a}{2}\cos(2\Omega t - \phi) . \quad (8)$$

Notice that the terms of in the RHS of eq. (8) are time-invariant, and sinusoidal with the frequencies of 1 Ω and 2 Ω, respectively. Referring to the Fourier series of $g_z(t)$, the coefficients in eq. (8) can be denoted as a_0, b_1 and a_2, correspondingly. They are known as the Fourier coefficients. Substituting these Fourier coefficients back to eq. (7), the averaged input energy $w_{\mathrm{d},z}$ can be represented in terms of them, or

$$w_{\mathrm{d},z} = \sqrt{a_2^2 - a_0^2} . \quad (9)$$

On the other hand, one assumes that there exists a linear system that is considered to be equivalent to the system (1) and has the governing equation

$$\ddot{y} + 2\zeta_{\mathrm{EQL}}\omega_n \dot{y} + \omega_n^2 y = f(t) , \quad (10)$$

subjecting the same initial conditions. ζ_{EQL} in eq. (9) is considered as the equivalent viscous damping ratio that to be determined. Note that the forcing function and the undamped natural frequency of the systems in (1) and (10) are kept the same. Similar to the nonlinear system (1), the linear system (10) has the solution

$$y(t) = y_0(t) + y_{\mathrm{p}}(t) . \quad (11)$$

In addition, the steady-state of $y(t)$ is considered to have the form

$$y(t) \approx y_{\mathrm{p}}(t) = Y\sin(\Omega t - \phi_{\mathrm{L}}) , \quad (12)$$

where ϕ_{L} represents the phase angle due to the existence of the equivalent viscous damping. Applying the same modulation as that in the nonlinear one, the modulated responses of the equivalent linear system is

$$g_y(t) = f(t)\cdot y(t) = \frac{FY}{2}\cos\phi_{\mathrm{L}} - \frac{FY}{2}\cos(2\Omega t - \phi_{\mathrm{L}}) . \quad (13)$$

Again, the first term, which is time-invariant, in eq. (13) is closely related to the system damping. Analogous to eq. (8), eq. (13) can be expressed in terms of the Fourier coefficient notations as

$$g_y(t) = (a_0') - a_2' \cos(2\Omega t - \phi_{\mathrm{L}}) , \quad (14)$$

where the prime is added to indicate those coefficients are of the linear one. Correspondingly, the total dissipated energy of the EQL system can be written in terms of these coefficients as

$$w_{\mathrm{d},y} = \sqrt{(a_2')^2 - (a_0')^2} . \quad (15)$$

In order to establish the equivalent relation between the nonlinear and linear systems, which are respectively governing by eqs. (1) & (10), one considers the differences of their amplitudes are within some small tolerance ε, or

$$|z(t) - y(t)| \le \varepsilon . \quad (16)$$

However, eq. (16) may be generalized and equivalently written as

$$\left|\frac{1}{2\pi}\oint f(t)[z(t) - y(t)]\, dt\right| \le \varepsilon^* , \quad (17)$$

if the computation is carrying out in terms of complete number of cycles from the steady-states. Note, parameter ε^* in eq. (17) also represents a small tolerance similar to ε since $f(t)$ is a bounded periodic function. Furthermore, one may simply compute the differences between the square of these two dissipated energies. Therefore, using eqs. (9) and (15) to (17), one has

$$\left|(w_{\mathrm{d,z}})^2-(w_{\mathrm{d,y}})^2\right|= \left|[a_2^2-a_0^2]-[(a_2')^2-(a_0')^2]\right|\le\varepsilon^* . \quad (18)$$

To satisfy eq. (18), a sufficient condition can be easily found, or

$$a_2'\to a_2 \text{ together with } a_0'\to a_0 . \quad (19/20)$$

The both conditions must be valid in the same time. However, if one looks closely, the former condition simply indirectly suggests $Y\to Z_a$. That is to require the output amplitudes of the two systems be equal. In the meantime, the latter condition in eq. (20) implies that

$$\cos\phi_{\mathrm{L}}\to\frac{a_0}{a_2}. \quad (21)$$

Or, the condition is to force the two phase angles, ϕ and ϕ_{L}, be equal. Notice that if a_0 = 0 from eq. (21), ϕ_{L} = π/2 or 90°. Readers are referred to [23] for the detailed discussion in this case. The tangent of the phase angle for the linear system has been well known and can be expressed as [e.g., 20]

$$\tan\phi_{\mathrm{L}}=\frac{2r\zeta_{\mathrm{L}}}{1-r^2}. \quad (22)$$

Thus, one is able to solve the equivalent viscous damping ratio (ζ_{L}) from eq. (22) and obtain as

$$\zeta_{\mathrm{EQL}}(r)=\frac{(1-r^2)}{2r}\left(\frac{a_2}{a_0}\sqrt{1-(a_0/a_2)^2}\right) \quad \text{when } a_0\ne 0, \quad (23)$$

where $r=\Omega/\omega_n$. Using eq. (9), equation (23) is in fact equivalent to

$$\zeta_{\mathrm{EQL}}=\frac{(1-r^2)}{2r}\left(\frac{w_{\mathrm{d,z}}}{a_0}\right). \quad (24)$$

Thus, if one has directly measured the dissipated energy from the nonlinear system, only one coefficient a_0 is enough. In addition, only the positive value of eq. (24) shall be taken in accordance with the definition of the damping ratio.

3. The Fourier Coefficients

The relation in eq. (23) or (24) depicts that the equivalent damping ratio of the EQL system has been expressed in terms of the Fourier coefficients or the dissipated energy of the original nonlinear system. Now, the central problem becomes how to correctly evaluate these coefficients so the relation can be properly applied. Notice that the first Fourier coefficient, a_0, is time-invariant, which is dc of the modulated signal. The easiest way to get it is to design a low-pass filter (LPF) and to get rid of the signals of those periodic terms from $g_z(t)$. Therefore, one obtains

$$a_0=\left\|g_z(t)\right\|_{\mathrm{LPF}}=\frac{F\cdot Z_a}{2}\cos\phi , \quad (25)$$

where ||...|| denotes the filtered value from the RHS of eq. (8). Alternately, one may simply compute the following integration

$$\frac{1}{L}\int_{L_0}^{L_0+L}(\mathrm{RHS})dt=\frac{1}{L}\left[\frac{FZ_a}{2}\int_{L_0}^{L_0+L}\cos\phi\, dt + Z_0F\int_{L_0}^{L_0+L}\sin(\Omega t)dt - \frac{FZ_a}{2}\int_{L_0}^{L_0+L}\cos(2\Omega t-\phi)dt\right] = \frac{FZ_a}{2}\cos\phi \quad (26)$$

in which all terms in RHS are zero except the first time-invariant one if L is taken in terms of complete cycles of Ωt. Practically, one can compute the value of the LHS of eq. (8) from the measured signals $g_z(t)$ in the same way as of the RHS, i.e.,

$$a_0=\tfrac{1}{L}\int_{L_0}^{L_0+L}g_z(t)dt\cong\tfrac{1}{n}\sum_{j=1}^{n}g_z(j\,\Delta t_s), \quad (27)$$

where Δt_s = $(1/f_s)$ and f_s is the sampling frequency, n the number of sampling points in L. In fact, a_0 in eq. (27) is the average value or the dc offset of the modulated signal $g_z(t)$ in $[L_0, L_0 + L]$. During the averaging, one may just takes L large enough, instead of caring about if the sampling is in complete cycles. By doing so, the error is bounded and has been proved in [21]. And, the longer L one takes, the smaller error reaches.

The evaluation of the coefficient a_2 is not as easy as its counterpart a_0. However, one still has a very useful tool available. Consider the Fourier transform of eq. (8), one has

$$\frac{F_T[g_z(t)]}{2\pi} = a_0 \cdot \delta(\omega) + i(\frac{b_1}{2})[\delta(\omega+\Omega) - \delta(\omega-\Omega)] + (\frac{a_2}{2})[\delta(\omega+2\Omega)e^{-i\phi} + \delta(\omega-2\Omega)e^{i\phi}] \quad ,(28)$$

where $F_T[.]$ denotes the Fourier transform from t into ω, $\delta(.)$ the delta (or impulse) function. Therefore, the first way to obtain a_2 is from the Fourier transform of $g_z(t)$. One has to identify the value at 2Ω which closely relates to a_2 based on eq. (28).

Examining equation (28), it is also possible to design a band-pass-filter (BPF) that is similar to eq. 25. That is, to obtain a_2 by a BPF

$$a_2 = \|g_z(t)\|_{\text{BPF}}, \quad (29)$$

with its central frequency at 2Ω. Figure 1 graphically depicts the idea.

It is also worthy to emphasize that the aforementioned linearization procedure is actually based on the equivalence of the dissipated energies. Or, the accuracy of the equivalent viscous damping ratio in eq. (24) is completely depending on how accurately $w_{d,y}$ can represent $w_{d,z}$. This also means that the ratio of $w_{d,z}$ to $w_{d,y}$, or

$$S_{\text{EQL}} = [w_{d,z}]/[w_{d,y}] \quad (30)$$

can be used as the quality index to signify that how well the nonlinearly damped system being represented. And, the ratio index is unity if the two systems are precisely identical. If S_{EQL}>1.0 implies the present EQL method under-estimates the original dissipated energy, or $w_{d,y} < w_{d,z}$.

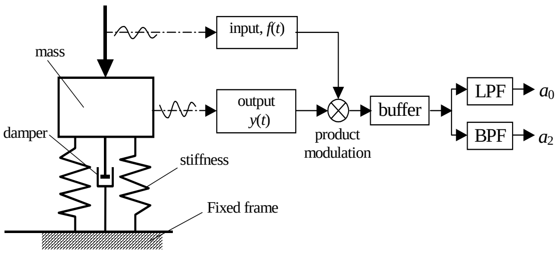

Fig. 1. Schematic diagram for obtaining Fourier Coefficients.

4. Numerical Examples

In order to verify the validity of the present EQL method, numerical simulations have been conducted. The nonlinearly damped system is considered to possess a bi-linear damping, or $\zeta = \zeta_1$ if $|\dot{z}| \le Z_c$ and $\zeta = \zeta_2$ if $|\dot{z}| > Z_c$, as shown in Fig. 2. The non-linearities of simulations can be controlled by specifying the two different damping ratios. In addition, in case of $\zeta_1 = \zeta_2$, the system would become a linear one with the viscous damping ratio ζ_1. During the simulations, the following parameters are employed:

- $\omega_n = 10\,\pi$ (or 5 Hz)
- $\Omega = 0.95\ \omega_n$ (or 4.75 Hz)
- $F = 10$
- $\dot{Z}_c = 1.0$

with consistent units. And, the sampling frequency is set to 200 Hz.

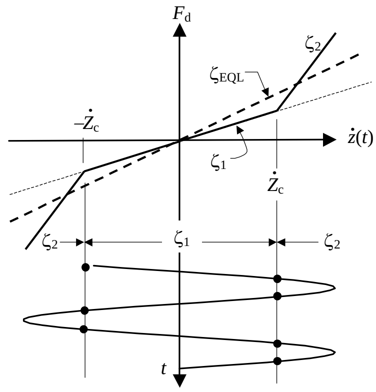

Fig. 2. A nonlinearly damped system with a two-sided, bi-linear damping.

After the EQL method being checked with the case of $\zeta_1 = \zeta_2$, several bi-linear simulations are followed. However, In order to further delve into the results predicted by the current EQL method, a typical time response is presented in Fig. 3(a). This figure was selected because that $\zeta_2/\zeta_1 = 3$ and the quality index, S_{EQL}, is approximately 1.1 or the original system dissipated 10% higher energy than the EQL one. As it can be clearly seen from Fig. 3(a), the time domain response indicates that the EQL system response (shown by solid line) somewhat smaller than the original nonlinear one (dotted line). Actually, this has already been indicated by the quality index. $S_{\text{EQL}} > 1$ is equivalent to the new EQL method under-estimating the average dissipated energy. The phase portraits of Fig. 3(a) are shown in Fig. 3(b), in which the portrait of the EQL system has been completely enclosed within that of the original one. As the consequence, the amplitude of $y(t)$ is smaller than that of $z(t)$. However, the phase angles of the both systems precisely coincide to each other even from the beginning of the transient response, shown in Fig. 3(b). In other words, this method has correctly forced the EQL system to have the same phase angle as the original one. But, on the other hand, the amplitude of the EQL system may be somehow sacrificed for the sake of $w_{d,y} < w_{d,z}$.

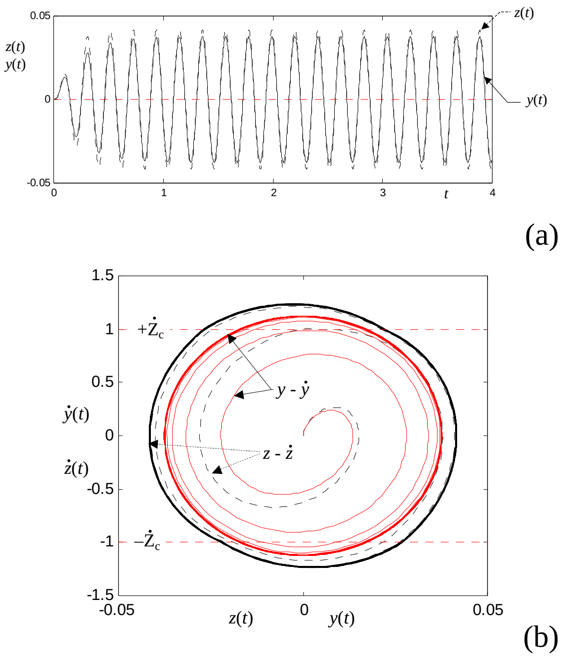

Fig. 3. Typical $z(t)$ and y(t) for ζ_1 = 0.05, ζ_2 = 0.15, ζ_{EQL} = 0.1326, and S_{EQL} = 1.1: (a) Time history; (b): Phase portrait.

In order to further study the modulated signal, the fast Fourier transform (FFT) has been performed to the modulated steady-state response, $g_z(t)$, of the time responses shown in Fig. 3. The transformed result is shown in Fig. 4. It can be seen from the figure that there exist two high peaks located at ω equals to 0 and 2Ω. And, the former is the one that denoted by a_0, the latter one-half of a_2, refer to Eq. (28). Not surprisingly, this result is exactly the same as that given in eq. (8). Meanwhile, this also indicates that the appropriate LPF and BPF from $g_z(t)$ are feasible to get a_0 and a_2. However, one cannot see any peak at Ω since Z_0 in eq. (8) is zero or too small to be seen. Besides, z(t) in Fig. 3(a) does not appear dc offset either.

In case the nonlinear system is altered by relaxing the nonlinear damping to one-sided bilinear. That is, the system damping is changed to $\zeta = \zeta_1$ when $\dot{z} \le Z_c$ and $\zeta = \zeta_2$ when $\dot{z} > Z_c$. The FFT result of the modulated signals is shown in Fig. 5. Comparing Figs. 5 with 4, the change of damping has accompanied with increasing in a_0. In the meantime, the peak located at Ω is now clearly seen. The height of this peak is one-half of b_1.

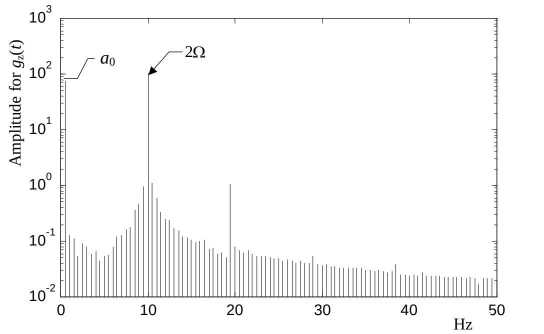

Fig. 4. Typical FFT of $g_z(t)$ for systems with two-sided bilinear damping.

As one has noticed that the coefficient b_1 does not play any role in the EQL procedures. However, it can be used as an index to signify how strong the nonlinearity can bring the system away from zero mean. The larger b_1 indicates that the system will behalf a stronger nonlinear characteristics and have a larger dc offset, refer to equation (8). Figure 6 substantiates this argument in which the dc offset Z_0 can be clearly seen. Obviously, there is no response dc offset for the EQL system response $y(t)$ since the external excitation has the zero mean and the system is linear. However, a non-zero response may exist for a nonlinearly damped system even though the excitation has a zero mean. This dc offset becomes zero if $\zeta_1 = \zeta_2$, which is linear.

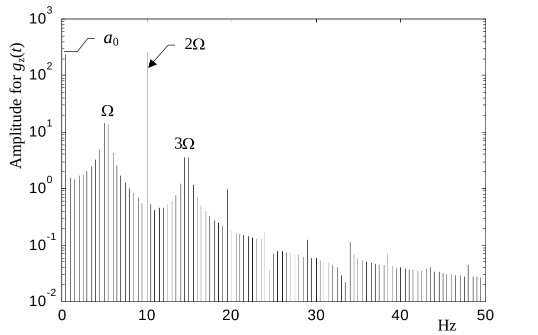

Fig. 5. Typical FFT of $g_z(t)$ for systems with one-sided bilinear damping.

5. Experiments

In addition to the numerical verification given in the last section, several experimental measurements have been carried out. The main objective of the experiment is to verify the validity of equation (8). The experimental setup for the measurements is shown in Fig.

7. The test specimen was a steel cantilever of dimensions $1.0^T \times 21.5^W \times 194^L$ mm (ca. 35.6 g) that was directly mounted on to a shaker. At the end of the beam, there was a plastic straw with the length 96 mm and the outer diameter 8.6 mm (ca. 2 g) firmly glued on to the beam. Meanwhile, in order to simulate damping forces acting on the beam, the straw was inserted into a cup of viscous liquid during the experiments. The inserted depth was set in such a way that the straw is just about to leave the liquid surface when the beam oscillates at its highest point. The damping forces can be intuitively imaged as nonlinear since the depth inside the viscous liquid is various with respect to the beam oscillations. Refer to Fig. 7 the detailed setup.

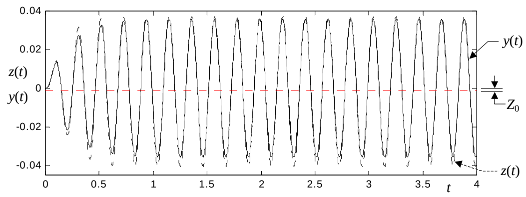

Fig. 6. Typical $z(t)$ and $y(t)$ of an one-sided bilinear system for $\zeta_1 = 0.1$, $\zeta_2 = 0.2$, $\zeta_{EQL} = 0.1402$, and $S_{EQL} = 1.09$.

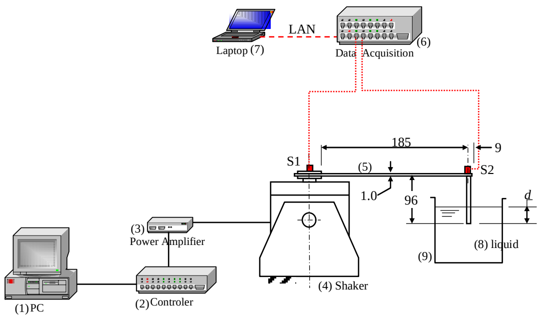

Fig. 7. The experimental set-up (1: PC; 2: signal controller, Dactron; 3: Power Amplifier, B&K 2706; 4: Exciter, B&K 4809, 5: Cantilever; 6: Acquisition system, iMC μ-Musys; 7: PC and Software, FAMOS, 8: Liquid, and 9: Glass cup).

During the experiments, the sampling frequency f_s was set to 200 Hz. And, sinusoidal excitations with the frequency were applied from the controller to the shaker and detected by accelerometer 1 (S1), which directly mounted on the base of the cantilever. The system responses were then detected and acquired by accelerometer 2 (S2), which was located at the tip of the beam. The both signals were then acquired and sent to the laptop computer for the later analyses.

Figure 8 shows the typical result in which the modulated signal has been transformed into the frequency domain. It clearly appears a peak at Ω in the plot. In fact, the height of that peak in the frequency domain can be realized as one-half of the coefficient b_1, refer to eq. (8). However, care should be taken when measuring b_1 from the plot since there exist also one other possibility affecting the peak at Ω. Experimentally, the digital filter that is applied during the data acquisition may also create an extra peak at the frequencies like $1\times\Omega$. This phenomenon is also known as the signal leakage [24]. Nevertheless, the experimental result has further verified the validity of eq. (8) which is the core of the present EQL method.

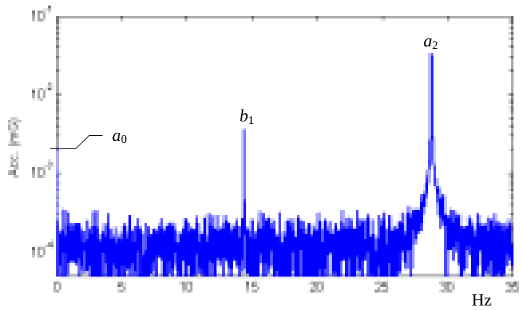

Fig. 8. A typical modulated response in the frequency domain.

6. Conclusions

A novel equivalent linearization method for nonlinearly damped and sdof systems has been developed in the present paper. The method is derived from modulating the steady-state responses of the original nonlinear system. During the linearization, the present EQL method requires the dissipated energy of the nonlinear system equal to that of the target linear one. As the consequence, this criterion is also equivalent to force both the phase angles and amplitudes of the two systems be equal or within a small allowable error. The equivalent viscous damping ratio can be then computed from the modulated signal.

In addition, the present report also theoretically explained the validity of the method. That is, this EQL method is *de facto* closely related to the Fourier coefficients of the modulated responses. Or, the dissipated energy of the nonlinear system can be expressed in terms of the Fourier coefficients. As the consequence, the core problem of this linearization method becomes how to correctly obtain these coefficients. There are a few ways have been discussed in the present paper, including averaging the signals, applying the filters and the FFT methods. The last has been clearly demonstrated and shown by the examples. The results of the examples indicate that the new linearization method gives the equivalent viscous damping with good accuracy. Moreover, carefully design experiments were carried out after the numerical simulations. The measured data also substantiate that finding the Fourier coefficients from the modulated signal is feasible.

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非線性阻尼結構系統之一種等效能損線性化法

黎文龍 及 曾百由

摘 要

本文提出一種新的等效線性化法，文中主要是找出線性系統之耗能與非線性阻尼者之相等。過程中，首先理論導出並計算原系統輸出反應之積調變，找出調變後訊號之傅立葉係數，再將能損以該係數表示，而等效之線性系統則透過該係數間之誤差最小化導出。本文中並以模擬及實驗證明本法之正確及有效性。

關鍵字：EQL、非線性阻尼、傅立葉係數