An Optimum Design Model for Down Acting Press Brakes of High Precision and Medium Span

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Keywords: Press brakes, Crowning system, Optimum Design

ABSTRACT

In the optimum design, it is very important to have a good mathematical model that can completely represent the real system. In the present paper, attention is given to provide a good mathematical model for a down-stroking press brakes. The objective function and the model are derived based on the modified elastic beam theory. Depending on the design requirement, the designer may select a dual, or three-force compensation system. All these possible choices are discussed in terms of non-dimensional parameters that readers can apply to design practice directly. In addition, the robustness of the individual compensation parameters of the model is also presented. The result of computation implies that the robust and optimal parameter set happens to be three-equal-force compensation. Finally, the given model is verified by a commercial FE package. It is shown that the results from the present model are in consistent with those obtained from the commercial package.

INTRODUCTION

A typical hydraulic press brake normally consists of two or four posts or a column frame, a moving ram, hydraulic actuators, a bed frame, a back gage set and a control unit. The motion of the ram includes down acting, up acting, and side acting, depending on its actuator configurations. The typical tonnage may vary from a few metric tons to over thousands while the working span may have a size more than eight to ten meters. In addition, the main tools of the press brakes include both punches and female dies. It is known that the punch which is held by the ram and the die on the bed frame play the key role in the process of sheet metal forming. However, from the viewpoint of the designer, it is natural to consider the precision of a press brake prior to that of tools.

Most research pays attention to sheet fabrication in forming processes instead of the machine itself. However, precisely describing sheet forming processes is not an easy task, essentially for the so-called visco-plastic or elastic-plastic process. That means, any bending process may involve deformation that governs greatly by the strain rate and thus the theoretical analysis is very difficult. As a consequence, a variety of analytical bending models have been proposed to address the relationship of stress and strain distortion to final shapes. For example, in order to simulate the springback of bends, visco-plastic models have been proposed by (Fu & Luo, 1995). In addition to the studies in these models, there are also researches in process planning that incorporate in either CAD (Forcelles, et al, 1996) or better sensing devices (Magee & De Vin, 1998) in order to obtain highly accurate products. More than that, there exist lots of reports in finite element (FE) simulations and modeling of the sheet development process studies (Ceglarek et al, 2001; Pourboghrat & Stelson, 1997). No matter if they are using commercial packages or non-linear models, the only aim is to find better processing conditions so that sheet products can be precisely controlled and predicted.

However, one obvious cause that has been ignored for a long period is the machine itself. As it is well known, the materials used for the construction of a press brake are not rigid even though the deformation is relatively small. However, as the requirement for precision is getting higher and the spans of machines are getting larger, this small deformation can never be

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ignored anymore. There exist several ways to minimize this down deflection of the bed frame. One is the so-called 'crowning device' in a press brake of high precision. For instance, it is possible to insert a wedge block between the punch and the upper ram to obtain an excessive down deformation on the ram. Or, to insert ones between the die and the bed, so that the bed deflection may be compensated by the excessive a priori upward deformation. Both ways are easy in construction. However, it is not that simple to adjust the wedges to the right deformation in operations. Therefore, several trials are usually required for a forming process.

An idea to adjust the angle of die during bending process is proposed here to eliminate the so-called 'longitudinal curvature' defect by Ona and Watari (1998). Actually, the idea is a way that focuses on tools to minimize the possible deflection in bed frames. The deformable punch developed by Zhang, et al. (1997) is also classified into this category. Another option is to add compensation forces to the bed frame by utilizing the hydraulic pressure.

The main objective of the present study is to provide a design model for optimizing the possible compensations for a down acting hydraulic press brake. Since the number of up acting compensation forces is limited to three, the investigation here is good for the maximum span approximate 3.5 to 4 meters, to the best of the author's knowledge. However, it is still valid for press brakes with their spans over 4 meters for three compensation forces. Beyond that range, readers are referred to Li et al. (2003). In addition, the study needs not be limited to air bending, even though the discussions are all based on this process. In case of the die bending process, the author believes that the study is also valid since the two processes basically differ in tooling instead of press brakes.

THE BASIC MODEL

Refer to Fig. 1, the upper ram of a down-acting press brake is supported or hung at its ends. The bending loads are delivered by the two actuators through the ram acting onto the sheet blank and the bed. That causes the sheet to bend into the required shapes. In the mean time, the bed frame under the die is correspondingly arc down with the workpiece since the bed is not rigid.

The designer of a press brake can provide several compensation forces acting up-ward to minimize the down deflection of the bed. Assume that the compensation force $\hat{F}$ can be modeled as a concentrated load with distance A away from the origin of the press brake that has span ℓ. And, the downward working pressure is uniformly acting to the fixed bed and may be looked as the distributed load w. Then, the configuration of the bed frame can be modeled as shown in Fig. 2. Thus, one can define the non-dimensional parameters as

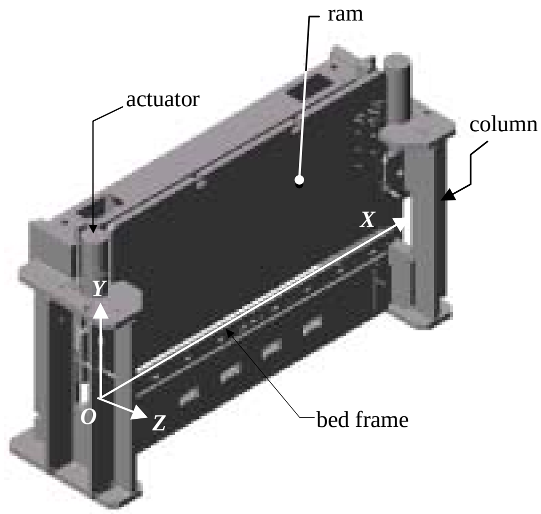


Fig. 1. A press brake and its coordinate system.

$a = A/\ell,\ x = X/\ell,$

where X is the distance measured from the origin. And the compensation force ratios are given as

$F_i = \hat{F}_i / (w\ell), \quad i = 1, 2\,.$

When the bed is under the uniformly distributed bending load w and the compensation forces F_1 and F_2, as shown in Fig. 2, the deflection of the bed at x may be approximated by

$$y(x) = \begin{cases} \alpha(F_1, a)\times y_{20}^L(x,a) + \alpha(F_2, 0.5)\times y_{20}^L(x, 0.5) \\ \quad + \alpha(F_1, 1-a)\times y_{20}^L(x, 1-a) - y_{10} \\ \qquad\qquad \text{if } \ 0 \le x < a \\ \alpha(F_1, a)\times y_{20}^R(x,a) + \alpha(F_2, 0.5)\times y_{20}^L(x, 0.5) \\ \quad + \alpha(F_1, 1-a)\times y_{20}^L(x, 1-a) - y_{10} \\ \qquad\qquad \text{if } \ a \le x < 1/2 \end{cases} \tag{1}$$

where the deflections are defined as

$$y_{10}(x) = \frac{Y(X)}{Y_{\max}} = 16\left[\left(\frac{X}{\ell}\right)^2 - 2\left(\frac{X}{\ell}\right)^3 + \left(\frac{X}{\ell}\right)^4\right], \tag{2}$$

$$y_{20}^L(x,a) = \frac{Y(X)}{Y_c} = \frac{\left(\frac{X}{\ell}\right)^2}{2\left(\frac{A}{\ell}\right)^3\left[1-\frac{A}{\ell}\right]}\left[3\left(\frac{A}{\ell}\right) - \left(1 + 2\frac{A}{\ell}\right)\left(\frac{X}{\ell}\right)\right], \tag{3a}$$

and

$$y_{20}^{R}(x,a)=y_{20}^{L}+\frac{1}{2\,a^{3}}\left[\frac{x-a}{1-a}\right]^{3}. \tag{3b}$$

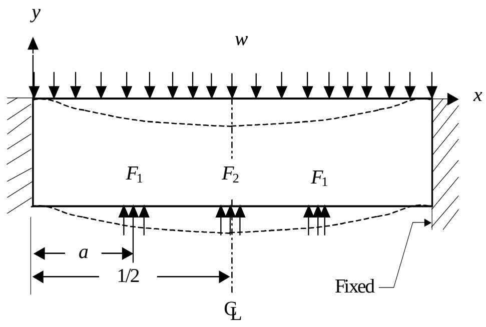


Fig. 2. Schematic diagram of the model for bed frames.

Also, the load parameter, α(.), is defined as

$$\alpha(F_i\,,a)=C_m\left(\frac{Y_c}{Y_{\max}}\right), \tag{4}$$

in which $C_m = C_k / C_u$ is the correction factor to tune the model deflection in such a way that their deviations are within an allowable level. The size of constant C_m strongly depends on the configuration of the bed and span ℓ. The modified central deflection, Y_c, of a fixed beam under a concentrated load $\hat{F}$ at distance A is represented by

$$Y_c = C_k\,\frac{\hat{F}\cdot A^3\cdot(\ell-A)^3}{3\,E\,I\,\ell^3}. \tag{5}$$

Hence, Y_{max} is the central deflection of the bed under a uniformly distributed load along the whole span is defined as

$$Y_{\max}=C_u\,\frac{w\cdot\ell^4}{384\cdot E\,I}. \tag{6}$$

The correction factors, C_k and C_u, must be evaluated in accordance with the geometry, configuration and material of the bed, as well as experiences of the particular manufacturer. Readers may be referred to Chen (2003) for the detailed discussions. For now, they are just set to be unity.

THE OBJECTIVE FUNCTION

In order to evaluate the deflected shapes, one has to define an evaluation function that well represents the model. Since the bed may be over compensated, as refer to Fig. 3, one can define the function as the square sum of the deviations along the span as

$$\phi(F_1,F_2,a)=\int_0^{\ell} y^2(x)\,\mathrm{d}x\,. \tag{7}$$

It is clear that this value can be regarded as the deflection residual after a compensating or crowning device has been applied. Thus, the optimal compensation forces and their locations can be evaluated and determined by minimizing the objective function ϕ. The procedures are shown in the following sections.

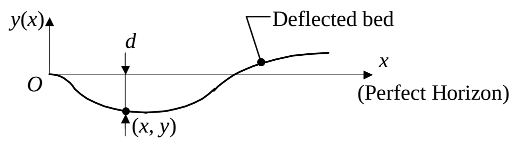


Fig. 3. Deflection of the bed along the x-axis.

OPTIMAL COMPENSATION DESIGN

For the sake of brevity without loss of the generality, the optimization techniques introduced here are limited to the unit correction factor C_m. For the case that C_m is not equal to 1.0, readers will find no difficulty to follow the procedure given below with one additional constant factor.

Dual Force Compensation

By setting $F_1 = F$ and $F_2 = 0$ in Equation (1), one has the case of two equal and symmetric compensation forces that straddle the mid-span centerline. Or, the objective residual function takes the form

$$\begin{aligned}\phi(F,a)=&\frac{512}{15}a^3F-\frac{768}{35}a^2F-2048a^5F^2\\&+\frac{3072}{5}a^4F^2-\frac{256}{35}a^8F-\frac{8192}{35}a^7F^2\\&+\frac{1024}{35}a^7F-\frac{2048}{5}a^8F^2-\frac{512}{15}a^6F+\frac{64}{315}\end{aligned}\,. \tag{8}$$

Clearly, the objective function in Eq. (8) has two parameters: the size of the compensation force F and its location a. By taking the derivatives with respect to these two parameters and set them to zero, one has

$$\frac{\partial\phi}{\partial F}=\frac{\partial\phi}{\partial a}=0\,, \tag{9}$$

The above expression stands for two different equations. Simultaneously solve the two equations for a and F and keep only those complying with the model, one has the global optimal solution pair as

$$F_{op} \approx 0.3447 \text{ at } a_{op} \approx 0.3257, \qquad (10)$$

with $\phi(F_{op}, a_{op}) = 2.956 \times 10^{-5}$. Figure 4 shows the deflection residual as the function of various compensation forces and their locations. The dotted line in the figure depicts that the most effective compensation force can have the cases that either F or a is to be decided. This line also reveals that F approaches one-half of the value the single force compensation or 0.26 as $a \to 0.5$. That is, the case of dual-force compensation reduces to that of the single mid-span compensation given in the last section.

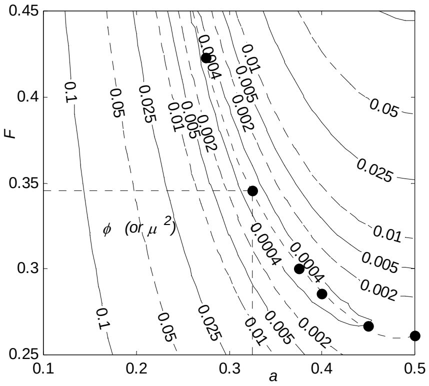


Fig. 4. The deflection residuals on the *a*-*F* plane.

The dotted line in Fig. 4 is very useful in the case of designing dual force compensation. For the readers' references, the compensation force on this line may be curve-fitted by a polynomial of a as

$$F(a) \approx -29.792a^3 + 38.3a^2 - 16.615a + 2.712, \qquad (11)$$

for $a \in [0.1, 0.5]$. The approximate deflection residuals in this curve are shown in Fig. 5. It clearly suggests that the dual force compensation around a < 0.2 is extremely ineffective. The optimal compensation location may be located in the region close to a = 0.32 ~ 0.33, as shown in Fig. 5.

Three-Force Compensation

For the case of three-force compensation, it is not as easy as its counterparts introduced in the former section. In order to simplify the optimization procedure, the materials here have been classified into two cases: (A) three forces are equal, or $F_1 = F_2$, and (B) un-equal forces, or $F_1 \neq F_2$. Both situations are very common in design practice.

(A) Equal Force Compensation. By setting $F_1 = F_2 = F$ in Eq. (1), one can treat the three-force compensation case as a two-parameter problem. Thus, the objective residual function will be similar to Eq. (8). However, one has the global optimal solution pair as

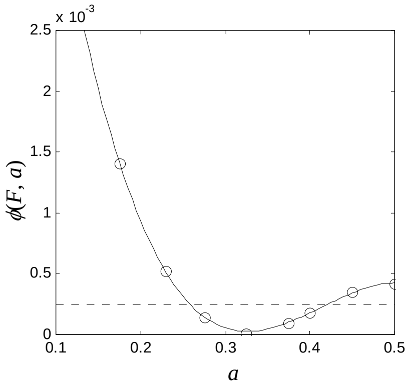


Fig. 5. The best delection residuals vs. a for dual force-compensations.

$$F_{op} \approx 0.256 \text{ at } a_{op} \approx 0.243, \qquad (12)$$

with $\phi(F_{op}, a_{op}) = 1.234 \times 10^{-6}$. Figure 6 shows that the deflection residuals as the function of compensation forces and their locations. And, the curve-fitted optimal compensation force has the form

$$F(a) \approx -2.47a^3 + 4.186a^2 - 2.368a + 0.62, \qquad (13)$$

for $a \in [0.1, 0.5]$ and is shown by the dotted line in Fig. 6. Similar to the case of dual force compensation, by supposing that the location of the compensation force approaches 0.5, as in reference to Fig. 6, the optimal force will approach 0.174 that is consistent with the case of single force compensation. In addition, the plot in Fig. 6 also indicates that the case of three-equal force compensation can reach better residuals than that of the dual force one. Thus, it is clear that this case is better in the sense of small residuals.

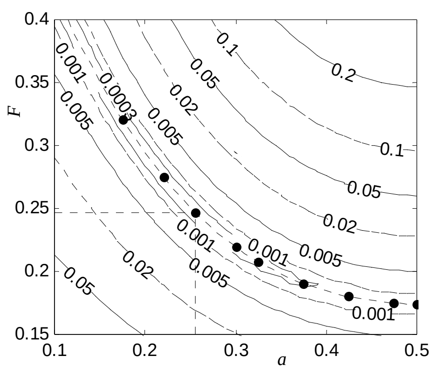
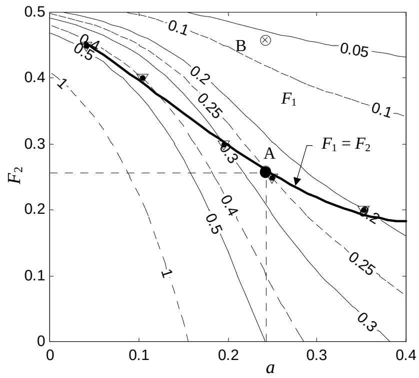


Fig. 6. The deflection residuals for three-equal-force compensation.

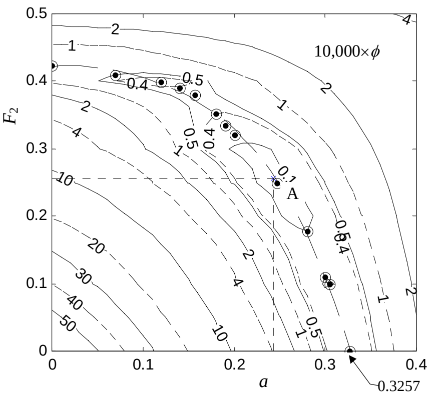
(B) Un-equal Force Compensation. A little more complicated case of three-force compensation is the case of un-equal compensation. That is, the case that $F_1 \neq F_2$, F_1 located at distance a away from the origin and F_2 is at the mid-span of the bed as shown in Fig. 1. Thus, the optimization procedure has to consider the three parameters: F_2 and F_1 together with its location a. Substituting Eq. (1) into Eq. (7) and performing the symbolic integration, one has the objective residual function or $\phi(F_1, F_2, a)$. Since it is too complicated, it will not be shown here. However, one may follow the same steps described in the last sections to evaluate all possible locations and sizes where minima exist. Similar to the former cases, the solutions of the possible minimum global residual can be obtained as

$$\begin{cases} F_{1,\text{op}} \approx 0.2554 & \text{at } a_{\text{op}} \approx 0.241 \\ F_{2,\text{op}} \approx 0.259 & \text{at mid-span} \end{cases} . \tag{14}$$

with the residuals of 1.23×10^{-6}. The result is shown by point A in Fig. 7(a). In the isometric residual plot, as shown in Fig. 7(b), the location of A is at the center of the minimum residual region.



(a)



(b)

Fig. 7. Three-force compensation: (a) Optimal parameters; (b) Residuals.

Both Figs. 7(a) and 7(b) have been purposely plotted in the same scale on the abscissa and ordinate. It is useful to over-lap these two plots for locating the three parameters and determining the optimal condition. By doing so, one may read values of F_1, a and F_2 as close as possible to or inside the region of the minimum residual. For example, it is impossible to get the smallest residuals within the residual region with values less than 0.1×10^{-5}, if one is to choose either $a \notin$ [0.2, 0.29] or $F_1 \notin$ [0.22, 0.29]. However, if one prefers to design a crowning force with residuals not greater than 0.5×10^{-5}, then the possible (a, F_1) combination options would be much wider since it occupies much more area than that of 0.1×10^{-5}, as referred to Fig. 7(b).

ROBUSTNESS OF PARAMETERS

Once the compensation forces and its location have been decided, the next issue is to check the robustness of these design parameters. In general, the variance of a design system can be obtained from the Taylor expansion of the parameters involved. That is, for a given set of independent parameters (F_1, F_2, a) with the function to be achieved, the standard deviation may be represented by the function (Li, 2000), or

$$\sigma_\phi = \sqrt{\left(\frac{\partial \phi}{\partial F_1}\right)^2 \sigma_{F_1}^2 + \left(\frac{\partial \phi}{\partial F_2}\right)^2 \sigma_{F_2}^2 + \left(\frac{\partial \phi}{\partial a}\right)^2 \sigma_a^2} \tag{15}$$

where the terms in the parentheses are based on the nominal values of the parameters, and their variances $(\sigma.)^2$. Furthermore, if the manufacturing tolerance implies three times of its standard deviation, or $t = 3\sigma$, then Eq. (15) is also valid for the tolerances. Therefore, the final system tolerance of ϕ is the sum of the products of the gradients of independent parameters and their corresponding tolerances. Besides, it is also clear that the quality of y is closely related to ϕ. Thus, ϕ can be regarded as the quality index for the bed deflection. The smaller deflection is, the better quality level should be.

Table 1. Deflection quality ϕ for different selections.

Choice / parameter	I	II	III	IV
F_1 ±0.01	0.255	0.256	0.290	0.255
F_2 ±0.01	0.259	0.256	0.200	0.255
a ±0.01	0.241	0.243	0.252	0.241
$\sqrt{\phi}\times10^3$	1.11	1.11	4.47	4.46
$\left(\sigma_{\sqrt{\phi}}\right)\times10^4$	0.32	0.05	1.35	1.58

Four different quality levels are given in Table 1. In case that one considers a set of (F_1, F_2, a) with nominal values at Choice I, which corresponds to point A given in the last section, and with manufacturing tolerances of ± 0.01, then the final bed deflection has the quality index of $(1.11 \pm 0.032)\times10^{-3}$, as shown in Table 1. On the other hand, if one selects Choice II, which is the optimal set for the three-equal-force compensation, it becomes $(1.11 \pm 0.005)\times10^{-3}$. This means that the parameter set II is more robust than that of Choice I. Therefore, the best choice of the bed compensation is Choice II even though they have the same quality index value of ϕ. In addition, for the reason of comparison, two more cases of different parameter combinations with the same quality indexes are also presented. They are denoted by Choice III and IV in the table.

In order to further investigate the sensitivities of the parameters to the final quality index, one may look at the partial contributions of each parameter in Eq. (15). In addition, one assumes that the three parameters have the same manufacturing tolerance, i.e., $\Delta F_1 = \Delta F_2 = \Delta a$. Therefore, the deflection quality is completely determined by the three partial differential terms: $(\partial\phi/\partial F_1)^2$, $(\partial\phi/\partial F_2)^2$ and $(\partial\phi/\partial a)^2$. Thus, in Choice I, for example, one starts computing the values of all these terms with set F_2 = 0.259 for the whole a-F_1 plane. By examining the resulting values, one can easily identify the parameter that contributes the most to the final quality. The most sensitive parameter is the one that has the largest contribution. Figure 8 shows the computed results on a-F_1 plane.

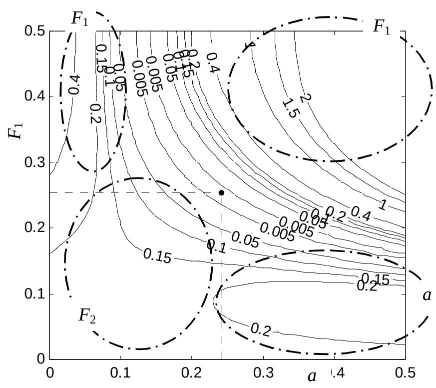


Fig. 8. The partial residual contributions on (a-F_1) when F_2 = 0.259.

Examining Fig. 8, one may find this residual sum plot be divided into four zones and each zone governed by one parameter. These zones are most sensitive to the parameter that governs the area in a-F_1 plane. However, there appears to be no parameter that controls the smallest residual region. Therefore, it is the author's suggestion to choose the design parameter inside the smallest residual zone so that it has more chance to obtain a robust parameter set

CAE VERIFICATIONS

The aforementioned optimization model and procedures can be verified by the CAE package. The commercial FEM package used here is the COSMOS/ DesignStar. The press brake of span 4 m and 100 metric tons is considered in the FE analysis. The material used for the bed frame was a low alloy steel that complies with JIS SM 490C with E = 2.1×105 MPa and ν = 0.28 with the area moment inertia I = 3.1×109 mm^4. In addition, by now the correction factors in Eqs. (5) and (6) are C_m = 1.0 and C_u = 2.0, respectively.

In order to see the effect of location a to the final residuals, the analysis starts by letting $\hat{F}_1 = \hat{F}_2 = 25.5$ tons (or F = 0.255) and the effects of locations at A = 960 and 965 mm, equivalent to non-dimensional locations 0.24 and 0.241, respectively. Note that the latter also corresponds to Choice IV in Table 1. The results are plotted in Fig. 9. It is clearly shown in Fig. 9 that the two results are in good agreement. The general trend is that the results from the proposed model are somewhat smaller in deflections than those of the FEM analysis. However, the present model predicts very well the effect of compensation location along the whole span.

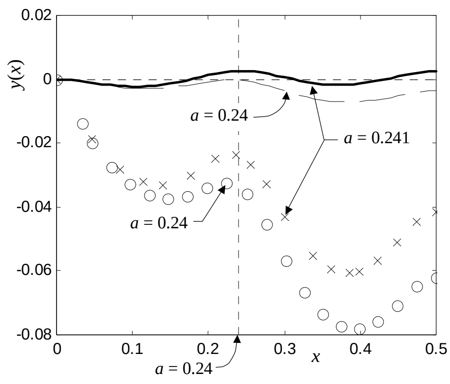


Fig. 9. Comparisons for the proposed model (Lines) & the FEM (Markers) for two different locations of a when $F_1 = F_2 = 0.255$.

On the other hand, the influence of the central compensation force to the deflection is also one major concern. In case that F_1 is kept the same both in size and location while F_2 varies from 0.255 to 0.259 in the analyses, the results are given in Fig. 10. Similar to Fig. 9, the predicted results of the present model reveal good consistency with those of the FEM analysis. However, the former estimates the compensation effect of F_2 relatively higher than it should be, so that the y computed from the FEM is smaller than that of the model. The possible reason may be due to the effect of concentrated compensation forces. It is inherently not feasible to have a concentrated compensation in real engineering practice. Besides, the geometric configuration of the bed also requires openings to hold the hydraulic actuators. Such openings obviously reduce the rigidity of the bedplate. Nevertheless, the difference may be corrected or tuned by incorporating an appropriate value of C_m.

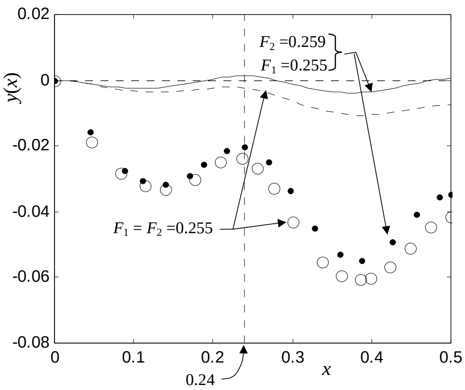


Fig. 10. Comparisons for the present model (Lines) & the FEM (Markers): (1) $F_1 = F_2 = 0.255$ and (2) $F_1 = 0.255$ & $F_2 = 0.259$. F_1 located at $a = 0.241$.

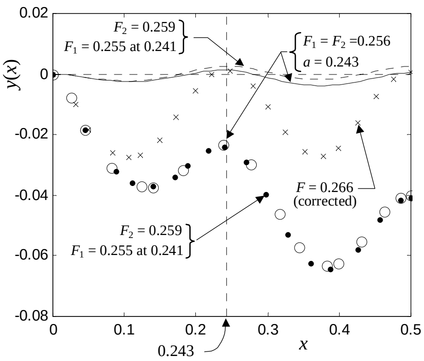


Fig. 11. Comparisons for the proposed model (Lines) and the FEM (Markers).

In case that the two optima (Choice I and II in Table 1) of three-force compensation mentioned in the last section are further investigated. It is found again that the present model gives the over-estimates on the compensation effects and hence the computed deflections are smaller than those of the FEM solutions. This is similar to that of Fig. 9 or 10. In case that the correction factor C_m is considered in the model, the over-estimation can be surely alleviated. For example, the corresponding tuned compensation force for Choice II is 0.266, or approximate 104% of the original one. Put the new compensation force back to the FEM package and the result is also presented in Fig. 11 by the cross (x) markers, together with those of uncorrected Choices I and II. The quantitative improvement can be clearly seen from the plot. Therefore, the result with the correction factor in this figure further substantiates the validity of the present model.

CONCLUSIONS

A mathematical model for design optimization on a compensation device of the press brake is proposed in this paper. The model is based on the modified elastic beam theory. By giving two correction factors, a general form and an objective function are derived. Depending on the design requirement and the spans, the designer may select a two-, or a three-force compensation system. All these possible choices are discussed in terms of non-dimensional mathematical equations so that the readers can directly apply this model in design practice. In case that dual force compensation is preferred, then the optimal compensation size would be 34.5%, located at 34.8% of span. On the other hand, 25.6% of the actuating force is recommended for the best choice for three-force compensation. In addition, the robustness of the individual parameters for the system is also given. As a consequence, the robust and optimal parameter set happens to be three-equal-force compensation. Finally, the model is verified by a commercial FE package. The results obtained from the present model are consistent with those of the commercial package.

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下壓式高精密折床之
最佳化設計模型

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摘 要

確實反應系統行為之數學模型為最佳化設計所不可缺的，本報告針對下壓式油壓折床提出一良好的數學模型。該模型係以彈性樑理論為基礎，並配合修正係數來達成，依所需補償的方式不同，得直接採用本報告之無因次結果，選擇單一、二或三補償力，以達成精密折床之最佳設計。報告中，並以一般商用 FEM 作為驗證，結果發現本報告之模型與 FEM 相當一致。