A Parameter Identification Method for Cantilever Systems by Using the Dissipative Energy

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Abstract - The main objective of the present report is to derive method for identifying system parameters of a cantilever system that is modeled by a single-degree oscillator. Unlike other existing methods, the present method starts with giving a wide band, or chirp excitation to the target system, and to locate the damped natural frequency of the oscillator. Once the damped natural frequency is obtained, it is possible to find the frequency at which the phase lag is equal to 90°. From which, the external excitation frequency is then purposely changed to that frequency. In the meantime, the system dissipative energy or power needs also be recorded. The system parameters, both the mass and stiffness of the oscillator, are expressed in terms of the external frequency and the system damping. The former is close related to the damped natural frequency while the latter can be identified along with measuring the input power. Applying the formulations provided in the present paper, it is possible to recognize the system parameters correctly, mass and stiffness. The novel formulations were then numerically simulated using the Simulink toolbox of MATLAB. The simulation results clearly showed the current method can work with good accuracy. Following the numerical simulations, the experimental validation was also carried out. Even though the experimental results again verified the correctness of the method, a larger error may exist if system damping is extremely small. The reason may mainly attribute to the measurement accuracy for dissipative power and the location error from the frequency of 90° phase lag. Nevertheless, both numerical simulation and experimental results strongly suggest that the new recognition algorithm can be applied with confidence. In addition, to the best of the author's knowledge, it is the first attempt to give such an almost-exact formulation for a cantilever system. More importantly, the method can be further generalized to other oscillator without difficulty.

Keywords - Parameter identification, Signature analysis, sdof oscillator, Dissipative energy.

I. INTRODUCTION

Lots of engineering practices still demand identifying system parameters to support calibration or to validate mathematical models especially at design stages. In addition, those recognition methods can also be used in damage or fault detection, optimal control, system health monitoring or diagnoses during systems are in service stages. As a consequence, methods of recognizing or identifying the system parameters have been popular topics for years for many researchers. For example, [e.g., 1, 2] studied vibration signals obtained from gearboxes and applied the so-called "detrended-fluctuation analysis" to determine the conditions of gearboxes. Unlike most researchers who focused on the evolution of statistical parameters [e.g., 2, 3], [1] using fractal properties of time series recorded from the gearboxes, it was able to distinguish the signals with respect to the working conditions. Similar to defects of geared systems, recognition or detection of cracks are also popular topics in identification studies. Instead of a single crack [e.g., 4, 5], problems of multiple cracks have been studied and existing methods were reviewed and documented in [6]. However, still this area is being paid more attention particularly from the fault identification and condition monitoring point of view.

As it is well-known that the methods for identifying system parameters may include both the time-domain and the frequency-domain approaches [7, 8]. The general advantage of the former techniques stems from directly evaluating the experimental or field data without transforming it into the frequency domain. However, the disadvantage may be that there exists no way to the frequency band of interest, while filtering process is normally needed. Unlike its counter part in the time domain, the approaches in the frequency domain need to perform transformation of data, mostly using the Frequency Response Function (FRF). Then, one may proceed either iterative or direct ways to extract the system parameters from that measured FRF. Furthermore, the frequency approaches may also be generalized to multi-degree-of-freedom (dof) by applying the concept of modal identification. In that case, it may be able to extract all the modal properties of the system. However, the main difficulty associated with these methods is that hardly to simultaneously obtain wide band responses, which contains many modes.

All these recognition methods, either for sdof or mdof, have their important contributions to some extent. However, they can be hardly applied to a small system of size of order 100 μm. Such systems are common for instru-ments in nano-technology like atomic force microscopes (AFM) [9]. Under such circumstances, one needs only parameters at the first mode, or at most a few modes, but with accurate results. In addition, system damping plays a very important role. Recently, the author has developed a novel algorithm to identify system damping [10] for base-excited systems. It has been further generalized to recognize the system parameters in this report. Even though the study is for linear sdof systems, it is belief that it can be generalized to mdof according to modal analysis. More importantly, to the best of the author's knowledge, it is the first attempt to give such an almost-exact formulation for system parameters.

II. DERIVATION OF THE METHOD

Lots of practical applications have been demonstrated by a sigle-dof system. For example, [11, 12] modeled an AFM probe at its clip and studied stiffness properties of probes. It has been found that single-dof system approach can have quite acceptable results, as far as the first mode is concerned. Motivated by this, the present report gives a complete and novel recognition method for a cantilever beam system under base-excitation in the sequel.

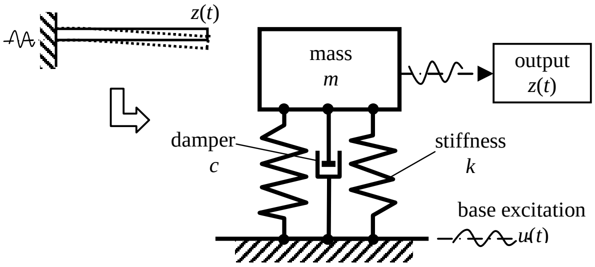

Fig. 1 A cantilever oscillator is modeled by a base-excited system.

A. System Mass and Stiffness

Fig. 1 shows the schematic diagram, together with notations, of a commonly seen base-excited, sdof system, which is the target system model in the present study. Let $u(t)$ be the input excitation from base, $z(t)$ the response of the system mass m, then the system governing equation can be represented by

$$m\ddot{z} + c(\dot{z} - \dot{u}) + k(z - u) = 0 \ , \tag{1}$$

where m, c, and k are the mass, damping and stiffness of the system, respectively. In case that one writes the input displacement $u(t)$ as

$$u(t) = Ue^{i\Omega t} \ , \tag{2}$$

then the mass response $z(t)$ can be expressed in terms of complex as

$$z(t) = Ze^{i(\Omega t - \phi)} \ , \tag{3}$$

in which ϕ is the phase angle between the input and output, and $i = \sqrt{-1}$. According to (2) and (3), it is obviously that if one may also define the damping ratio, frequency ratio and radial frequency in a commonly used way. Or,

$$\varsigma = \frac{c}{2\sqrt{mk}} \ , \quad r = \frac{\Omega}{\omega_n} \ , \ \text{and} \quad \omega_n^2 = \frac{k}{m} \ . \tag{4}$$

Using parameters in (4), $z(t)$ can be written in terms of its input $u(t)$. Or

$$z(t) = \left[\frac{1 + i(2r\varsigma)}{(1 - r^2) + i(2r\varsigma)} \right] U \ e^{i\Omega t} \ . \tag{5}$$

In the upper equation, Z is the amplitude of $z(t)$, and can be expressed in terms of the input amplitude U as well as the transmissibility T_z:

$$Z = T_z \cdot U \ . \tag{6}$$

That is, the transmissibility can be derived from (5) and defined as

$$T_z(r, \varsigma) = \left[\frac{1 + (2\varsigma \ r)^2}{(1 - r^2)^2 + (2\varsigma \ r)^2} \right]^{1/2} \ . \tag{7}$$

Notice that (7) T_z is function of r and ς in (7). In the meantime, one may also define the phase angle of (3) as

$$\tan\phi = \frac{m \ c \ \Omega^3 / \Lambda}{[k(k - m\Omega^2) + (\Omega c)^2] / \Lambda} = \frac{(2\varsigma \ r^3)}{[(1 - r^2) + (2r\varsigma)^2]} \ , \tag{8}$$

where

$$\Lambda = (k - m\Omega^2)^2 + (\Omega c)^2 = k^2 \left[(1 - r^2)^2 + (2r\varsigma)^2 \right] \ . \tag{9}$$

Obviously, from (7) and (8) that there is no way to compute phase angle ϕ unless one has the system damping c. Therefore, the central problem stems from the evaluation of system damping.

However, on the other hand, the denominator of (8) is actually equal to cos ϕ. Under this circumstance, the present method is to apply an external excitation with its frequency Ω in such way that the phase angle ϕ is purposely equal to 90°. That is, let the frequency where $\phi = \pi/2$ be ω_0, i.e., the method urges the experimenter to set $\Omega = \omega_0$ in (8). Thus, the denominator of (8) also has to satisfy

$$(1 - r_0^2) + (2r_0\varsigma)^2 = 0 \ , \tag{10}$$

where $r_0 = \omega_0 / \omega_n$. Or, equivalently

$$\frac{\omega_0}{\omega_n} = \frac{Q}{\sqrt{Q^2 - 1}} \ , \tag{11}$$

since

$$\omega_d / \omega_n = \sqrt{1 - \varsigma^2} \quad \text{and} \quad Q = \frac{1}{2\varsigma} \ . \tag{12}$$

Here, one has defined ω_d is the so-called damped natural frequency, which can be experimentally measured by e.g., a chirp or wide-band excitation. Using (11) with (12), one obviously can come to a conclusion [10] that the three frequencies satisfy:

$$\omega_d \le \omega_n \le \omega_0. \tag{13}$$

On the other hand, because the present method requires the frequency of the external excitations to be exactly at ω_0, the phase angle must satisfy $\tan\phi \to \infty$. Equivalently, it mathematically means the denominator of (8) must be zero, together with the numerator of an arbitrary non-zero constant. Therefore, one writes

$$\begin{cases} \dfrac{m \ c \ \omega_0^3}{(k - m \ \omega_0^2)^2 + (\omega_0 c)^2} = \sqrt{\alpha} \\ k(k - m \ \omega_0^2) + (\omega_0 c)^2 = 0 \end{cases} \tag{14}$$

in which α represents any non-zero, real constant. Solving (14), one at least can have a set of solution that satisfies

$$k = \frac{c \cdot \omega_0}{\sqrt{\alpha}} \quad \text{and} \quad m = \frac{c(1+\alpha)}{\omega_0 \sqrt{\alpha}} \ . \tag{15}$$

By using (4) which is the definition of the natural frequency or the ratio of k to m, one follows that the arbitrary constant α in (14) must be

$$\alpha = \frac{1}{Q^2 - 1} \ . \tag{16}$$

Substituting this expression back into (15), one can recognize the system parameters k and m as a function ω_0:

$$\begin{cases} k(\omega_0) = c \cdot \omega_0 \sqrt{Q^2 \text{ -} 1} \\ m(\omega_0) = \dfrac{c}{\omega_0} \cdot \left(\dfrac{Q^2}{\sqrt{Q^2 \text{ -} 1}} \right) \end{cases} \ . \tag{17}$$

The formulation in (17) is exact and good for $\Omega = \omega_0$. To the best of author's knowledge, this has not been attempted in literature. Note that (12) for the quality factor Q is the only approximate form thorough the derivation. Therefore

(17) may be referred as an "almost" exact form. Using equation (12), ω_0 in the upper equation may be replaced by ω_d which can be directly measured. Thus, one may also write (17) in terms of ω_d as

$$\begin{cases} k(\omega_d) = c \cdot \omega_d \left(\dfrac{2Q^2}{\sqrt{4Q^2 - 1}} \right) \\ m(\omega_d) = \dfrac{c}{\omega_d} \cdot \left(\dfrac{\sqrt{4Q^2 - 1}}{2} \right) \end{cases} . \tag{18}$$

Unfortunately, notice that from either (17) or (18), one can experimentally or indirectly obtain all parameters on RHS except system damping c. Or, unless system damping is known, the two expressions, both involve one unknown, are not much help in identifying system parameters. Thus, one has to recognize the system damping prior to estimate the system mass or stiffness.

B. System Damping: Dissipative Energy

Theoretically, the total energy involves in every $z(t)$ cycle of oscillation may be obtained by integrating (1). Or,

$$\Delta E = \oint [m\ddot{z} + c(\dot{z} - \dot{u}) + k(z - u)] \cdot dz . \tag{19}$$

The O-integral in the upper equation denotes the integration is done for a complete $z(t)$ cycle. Replace dz by $\dot{z}\,dt$ in (19) and integrate from 0 to $2\pi/\Omega$, one may compute ΔE. For the reason of simplicity, one lets input excitation for (2) be

$$u(t) = U \sin(\Omega t) . \tag{20}$$

Hence, from (3) $z(t)$ becomes

$$z(t) = T_z U \cdot \sin(\Omega t - \phi) . \tag{21}$$

Furthermore, applying (20) and (21) to (19), one has the form

$$\Delta E = c\pi (T_z U)^2 \Omega + c\pi (T_z U^2) \Omega \cos\phi - k\pi (T_z U^2) \sin\phi , \tag{22}$$

for the total involved energy. Examining the upper equation, the third term on the RHS is strain energy from the relative motion of $u(t)$ to its response $z(t)$. The energy does not dissipated away when the response. Or, stiffness can only deliver the energy from the base to the end mass once the probe system is in its steady-state. In addition to this strain energy, the second term to the RHS is regarded as the damped energy from these relative motions. Similar to this one, the first term to the RHS also dissipates energy away due to the damping of the system. However, during steady-state, the energies of the two terms have to be externally made up so that the oscillation can go on. As the cones-quence, the total dissipative energy per cycle can be expressed by

$$\Delta E_d(\Omega) = c\pi (T_z U)^2 \cdot \Omega + c\pi (T_z U^2) \cdot \Omega \cos\phi , \tag{23}$$

in case of steady-state oscillations. To simplify (23), recall from (15) that one has set the external frequency $\Omega = \omega_0$. That is, $\cos \phi = 0$, or (23) can be further simplified as

$$\Delta E_d(\omega_0) = c\pi\omega_0 \cdot (T_z U)^2 , \tag{24}$$

for every steady-state oscillation cycles. Note that [13] has reported a way to measure the dissipative energy via the product modulations of the input and output signals. Besides, the input power (or energy) must be equal to the system dissipated energy ΔE_d during the steady-state. In other words, this makes it possible to measure the system dissipated energy externally. Thus, for now, one just re-arranges (24), and using the averaged dissipative power $\Delta p_0 = (\omega_0 \Delta E_d)/2\pi$, the system damping c can be solved as

$$c = \frac{2\,\Delta p_0}{(T_z\ U)^2 \cdot \omega_0^2} . \tag{25}$$

In addition, it is because that the response amplitude Z_0 is much larger than that of input U since $\Omega = \omega_0$ is quite close to the system resonance. One thus can more accurately measure Z_0 instead of U. Or, it is more reasonable to write (25) into the form

$$c = \frac{2\,\Delta p_0}{Z_0^2 \cdot \omega_0^2} . \tag{26}$$

Notice that all the parameters on the RHS of (26) are measurable. Obviously, the accuracy of parameter esti-mation depends on how accurate the three parameters on RHS can be measured. Nevertheless, damping c can be experimentally determined. Using the upper (26) to (17), the estimated system stiffness becomes

$$\hat{k} = \frac{2\,\Delta p_0}{Z_0^2 \cdot \omega_0} \cdot \sqrt{Q^2 - 1} , \tag{27}$$

while the mass

$$\hat{m} = \frac{2\,\Delta p_0}{Z_0^2 \cdot \omega_0^3} \cdot \left(\frac{Q^2}{\sqrt{Q^2 - 1}} \right) . \tag{28}$$

Therefore, one can conclude the parameter identifying steps from the above derivation as follows:

(1) Apply a chirp or wide-band signal to scan the location of the damped natural frequency of the target system as well as the quality factor of the system.

(2) Use the formulations given in (11) and (12) to compute the frequency location where the phase angle equals to 90°. Denote that frequency as ω_0.

(3) Apply external excitation again to the target system but with the frequency right at ω_0 .

(4) Acquire the steady-state responses, both input and output. In case power meter is available, it is recommended to measure the dissipative power simultaneously.

(5) Utilize the equations given above and compute the corresponding spring constant of the system.

(6) Repeat the former steps for checking and confirmation.

III. VERIFICATIONS

A. Numerical Simulations

In order to confirm the theoretical derivation in Section II, a single-dof Simulink model was established, as shown in Fig. 2. Following the steps, one started with finding the natural frequency of the system by applying a chirp signal to scan the damped natural frequency here. The following system parameters, with consistent units, were freely given: $m = 3$, $c = 5 \sim 100$ (equivalent to ca. $\varsigma = 0.1\% \sim 2.5\%$), and the system natural frequency f_n was set to 100 Hz. In addition, the sampling frequency f_s and sampling time T were 2 kHz and 5 sec, respectively, while it has assumed that the dissipative energy could be correctly measured. The results are given in Table 1 where the notations with overhat denote estimated values. It can be seen clearly from the table that the current identification method can correctly estimate the system parameters, with small amount of errors. Besides, system parameters of lightly damped systems appear to be more difficult to recognize. However, the main error source comes from the error percentage of Q, as shown in the table.

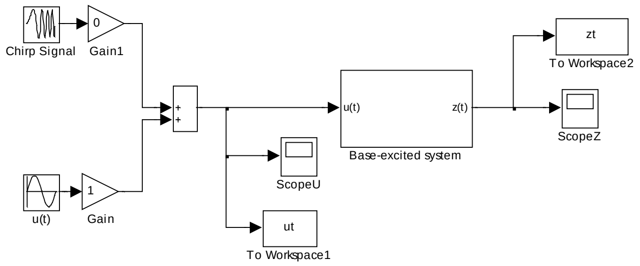

Fig. 2 The Simulink model for numerical verifications.

TABLE 1. Simulation results for m = 3, f_n = 100 Hz (T = 5 sec).

Damping		Mass (m)		Stiffness (k)		Quality factor (Q)		
c	ς (%)	$\hat{m}$	ε_m (%)	$\hat{k}$	ε_k (%)	Q_0	$\hat{Q}$	ε_Q (%)
5	0.13	2.839	-5.36	1.121E+06	-5.36	384.6	356.5	-7.30
10	0.27	3.002	0.05	1.185E+06	0.05	185.1	187.9	1.47
20	0.53	3.032	1.07	1.197E+06	1.07	94.34	94.33	-0.01
30	0.80	3.003	0.11	1.186E+06	0.12	62.50	62.93	0.69
50	1.33	3.009	0.30	1.188E+06	0.32	37.59	37.81	0.57
70	1.86	3.014	0.45	1.190E+06	0.48	26.88	27.04	0.59
100	2.65	3.019	0.62	1.193E+06	0.69	18.87	18.97	0.54

B. Experimental Measurements

In addition to the numerical verifications given in the last section, several experimental measurements were carried out. The main objective of the experiments is to verify the validity of (27) and (28). The experimental setup for the measurements is shown in Fig. 3. The test specimen is an austenite stainless steel (Young's modulus E = 210 GPa) cantilever with dimensions $1.6^T \times 20.2^W \times 250^L$ mm (ca. 64.31 g) directly mounted to a shaker. At the end of the beam, it is a plastic straw with the length 80 mm and the outer diameter 8.6 mm (ca. 0.2 g) firmly glued to the beam. Meanwhile, in order to simulate the damping forces acting on the beam, the straw is inserted into a cup of viscid liquid during the experiments. The depth of submersion is set in such a way that the straw is at the depth d when the beam is at its static equilibrium position. For example, in the case of d = 0 mm, the tip of the straw is just about to contact the liquid surface before the beam starts oscillating. On the other hand, the tip completely immerses in the liquid during the oscillations if d = 20 mm. Since the damping force is proportional to the depth inside the liquid, the straw thus acts as a damper in this experimental system.

During the experiments, the sampling frequency f_s was set to 200 Hz. And, sinusoidal excitations with the constant frequency (Ω) were applied from the controller to the shaker and detected by accelerometer 1 (S1), which directly mounted on the base of the cantilever. The amplitudes of these excitations were all kept the same while its frequency may be changed if so desired. The system responses were then detected and acquired by accelerometer 2 (S2, wt. 5.5 g), which was located at the tip of the beam ca. 240 mm from its fixed point. Both signals were then acquired and sent to a laptop computer for the later analyses.

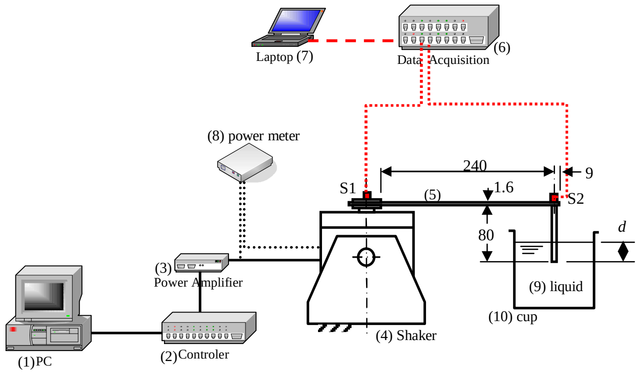

Fig. 3 The experimental set-up (1: PC; 2: signal controller, Dactron; 3: Power Amplifier, B&K 2706; 4: Exciter, B&K 4809, 5: Cantilever; 6: Acquisition system, iMC μ-Musys; 7: PC and Software, FAMOS; 8: Power meter; 9 & 10: damper).

Using d = 0 and no liquid in the cup, or there is no external damper, the equivalent flexural stiffness of the current test specimen, modelled as a cantilever beam, may be found from fundamental textbooks, or

$$k_t = \frac{3EI}{\ell^3} = \frac{3\times 210{,}000\times[\dfrac{20.2\times(1.6)^3}{12}]}{(240)^3} = 0.3142\ (\mathrm{N/mm})\,.$$

In addition, the deflections of the specimen were measured by putting on static loads at the location of S2 which is 240 mm from the fixed point (S1). From this deflection chart, the equivalent stiffness can be obtained: $k_s = 0.369$ N/mm. Having the information and set-up *a priori*, the experiments started carrying out. The beam specimen was excited by a chirp sine signals and found ω_0 located at somewhere between 12.7 and 12.8 Hz, refer to Fig. 4(a). Unfortunately, the lab did not have a controller to get any further accurate values than these. Therefore, the measures were completed at these two frequencies, along with a statistic analyzing tool. Note also that the so-called Fourier Coefficient method given in Sec. 2.3 has been adopted for the calculation of dissipative energy, cf. Fig. 4(b). From Fig. 4(b), notice also that the Fourier coefficients $\alpha_2 \gg a_0$ even though coefficient a_0 is not exact zero. In fact, coefficient a_0 here also plays as a role for checking whether ϕ is right at π/2. In case ω_0 is correctly located, coefficient a_0 should be very close to zero. Otherwise, an incorrect ω_0 is obtained. The time domain results, shown in Fig. 4(c), substantiates that ω_0 is quite close to 90°. The experimentally measured results are thoroughly shown in Table 2, together with the theoretically computed beam stiffness.

As it was mentioned earlier, the beam specimen has its total mass of ca. 64.5 g, while the identified mean is 58 (SD 5.6) grams. On the other hand, the identified stiffness has its mean at 0.372 (SD 0.0385) N/mm, as shown in the table. Consequently, the 90% confidence intervals (CIs) for the estimated mass and stiffness are ranged from 46.1 to 69.9 g and [0.290, 0.454] N/mm, respectively. In addition to the two means are close to one has expected, the both estimated mass and stiffness values are within their 90% CIs. However, the mass has a smaller error than stiffness. It is may be interesting in the error of the estimated mass using eqn (35). In fact, the 90% CI is [46.5, 69.7] with mean 58.11 g. Therefore, there exists only negligible difference between (28) and (35) as far as the recognition of mass is concerned. Notice also that the mean of the identified stiffness ($\hat{k}$) is very close to either the theoretical (k_t) or statically measured (k_s) ones. Nevertheless, it is clear that the present identification method can estimate the system stiffness and mass with very good accuracy. Without any doubt, the experimental results surely validate that the present method is an effective tool to identify the parameters of a SDOF, base-excited system.

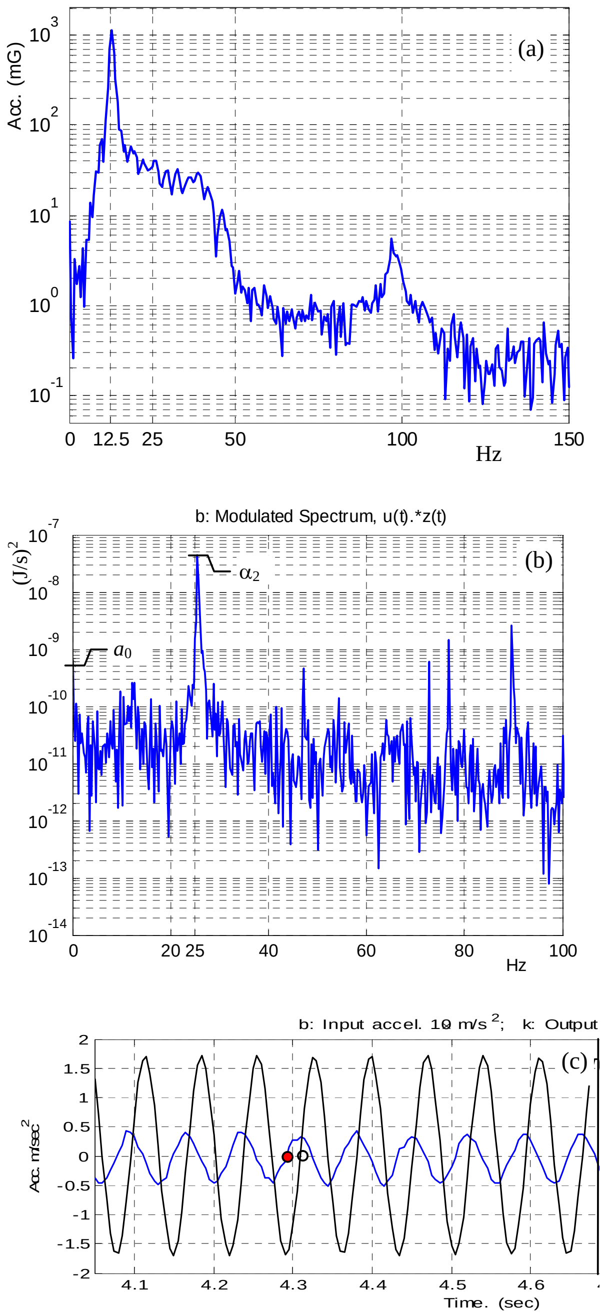

Fig. 4 Typical experimental results: (a) Frequency results from the chirp input; (b) Modulated signal where shows $\alpha_2 >> a_0$; (c) Input excitation and output response in the time domain.

TABLE 2 Comparisons of system parameters.

	Mass $\hat{m}$ (g)	Stiffness $\hat{k}$ (N/mm)	k_t (N/mm)	k_s (N/mm)
12.7 Hz	55.5	0.3524	beam model: $k_t = 3EI/\ell^3$	x: static deflections $F_{wt} = 0.369x - 0.233$
	53.4	0.3396		
	50.9	0.3235		
	51.4	0.3267		
	53.8	0.3420		
12.8 Hz	66.3	0.4273		
	62.1	0.4007		
	62.5	0.4029		
	63.3	0.4087		
	60.8	0.3920		
mean	58.0	0.3716	0.314	0.369
SD*	5.6	0.0385		

During the experimental measurements, it has been found that the major error sources are from the estimation of the quality factor Q and the frequency location ω_0. In addition, one other possible error source may be from the measurement of the dissipative power or energy which was not included in the present discussion.

IV. CONCLUDING REMARKS

A novel parameter identification method for single degree, viscously damped base excited systems has been effectively developed and presented in this paper. Unlike most existing methods, the present method starts with giving a wide band, or chirp excitation to the target system, and acquires the corresponding responses that are then used to scan and lock the damped natural frequency. Once the damped natural frequency is obtained, it is possible to locate the frequency at which the phase lag is equal to $\pi/2$ by using the formulation given in the present report. From which, the present method is to require the external excitation frequency purposely changes to that frequency and the steady-state responses are stored for the later analysis. In the meantime, the system dissipative energy or power may also be experimentally measured for estimation of the system damping.

In fact, the present identification formulation is to express both the mass and stiffness of the target systems in terms of two measurable parameters. They are the phase angle as well as its system damping. The former can be computed from the damped natural frequency and later used to locate the input excitation frequency of steady-state responses. On the other hand, the latter can be identified along with measuring the input power since the equivalence of the dissipated and input energies when the system is in steady-state. As long as these two measurable parameters are available, it is possible to apply them to the equations provided in the present paper.

After the theoretical derivation, the report also provided numerical simulations using the Simulink toolbox of MATLAB. Depending on the error of the quality factor, which is a function of the system damping, the simulated results clearly showed the current identification method can work with good accuracy. In addition, the results also revealed that it is more difficult to recognize the system if the system damping is small. The main reason stems from the formulation of the current method highly depends on the accuracy of damping. Given that the smaller error in damping, it is possible to have a more accurate in identifying the corresponding system mass and stiffness. Nevertheless, the numerical simulation results substantiated the validity of the method. Following the numerical simulation, experiments were also carried out by a cantilever beam system. The accurate experimental results once again confirmed the validity of the method, and the errors of the identified mass and stiffness are all negligibly small. Thus it is accordingly concluded that the present recognition algorithm can be applied with confidence.

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